A Stochastic Model for Energy Consumption Entailed by Mobile Device Proliferation

Submission by Team 5809

***Abstract*—Recent years have experienced a rapid transit towards personal mobile communication device possession. As such, stationary communication protocols such as landline telephones have experienced a steady decline of existence. In this work, we explore the socioeconomic impacts of one possibility entailed by the aforementioned trends, namely a complete replacement of landline communication devices by cell phones. We begin our analysis by examining the current statistics of cell phone and landline usage in the United States. We then describe one possible model that complete landline usage degeneration might demonstrate, following which we examine the corresponding impact on cell phone proliferation to balance the communication loss entailed by the former development. In our model for cell phone usage frequency over time, we closely consider a myriad of complexities that cause fluctuations in the number of cell phones in active usage at any given instance of time, and attempt to model them stochastically using simple Brownian motion. Using these usage models, we proceed with our primary analysis of the impact of cell phone replacement of landlines on the energy profile of the United States (and a similar) economy. Our analysis spans both energy consumption and ensued production costs, and also further addresses the impacts of population or economic growth as well as varying cell phone energy consumption rates on this energy profile. We culminate with model evaluation of our work, and discuss possible improvements to the same.**

***Index Terms*—Communication Devices, Energy Profile, Stochastic Differential Equations, Brownian Motion**

I. Introduction and Overview

In only 26 years, the cell phone has become so ubiquitous that it holds a designated spot in our hands, pockets, and purses. Teens, children, adults, and seniors have all been accustomed to the technological world of easy text messaging and quick phone calls without dealing with pesky wires connected to the corners in our houses and business, confining us to one location for conversations. McClatchy, of Tribune Business News states cell phones are not only useful for conversing but also to take photos, capture video worthy moments, and browse the Internet in need [25].

Currently, "there are more than 262 million wireless users in the U.S. alone, and the industry's annual revenues have topped $140 billion." Furthermore, "an entire generation has grown up using cell phones, an increasing number of consumers use them exclusively, going without a land line, and not even Superman bothers looking for a phone booth in which to change anymore [25]." In an internet survey conducted by JupiterResearch, it was found that 12 percent "do not subscribe to fixed voice service, and nearly two-thirds of them are ages 18 to 34." Although 70 percent of Internet users still possess landlines from a telecommunications company, 12 percent displayed their intent to replace home phone service with their cell phone providers in the next year [24]. Therefore, as more generations grow, it may be expected that cell phones will ultimately replace landlines forever.

Many have already planned a cell phone domination within the next few years where Doug Grant, director of wireless systems business development for Analog Devices, quoted, "by 2010, smartphones will represent 15% of all cellphones sold worldwide, and 75% of cellphones sold will be feature phones [25]." With new trends developing each day, it is inevitable to stray away from involving oneself in the international race to technological development. More and more, cell phones are outweighing the necessity of landlines where in countries such as South Korea, it can be used simultaneously to double as smart cards that can be waved in front of a vending machine to make a purchase, ridding infrastructure and the need to even carry around wallets. The consequences of cell phone usage far outweighs the benefits of landlines which cannot do anything but make a simple phone call. As there continues to be increasing development in the telecommunications area, cell phones are becoming surprisingly compact and multi-functional, indicating a new generation of technological advances in the future.

Our work in this paper attempts to capture an important aspect of this upward trend in cell phones, namely its displacement of landlines, and the subsequent energy consequences of the same. In the next section, we present our basic model for representing the energy change caused by a decline in landline communication and growth in mobile phone usage during the transition stage. This model accounts for various complexities in the quantity of cell phones in use, but presents only a simplified image of power consumption differences between cell phones and landlines. Within the next section, we use our developed model to formulate the optimal transition policy from landlines to telephones, and discuss energy consequences based on our model, as well as a broad overview of related outcomes. In Section 3, we extend this analysis to the steady state, after landlines have mainly been expunged from the system in context and replaced by cell phones almost entirely. We then present an improved model for energy consumption differences between cell phones and landlines in Section 4, that account for various device charging protocols, as well as wasteful practices in charging (such as unnecessary charging), and use this improved cost model to explore the energy consequences of these actions. We finally utilize our complete model in Section 5 for prediction of energy profile of the growing United States economy in the next few decades. We culminate with an evaluation of our model, and potential avenues of improvement.

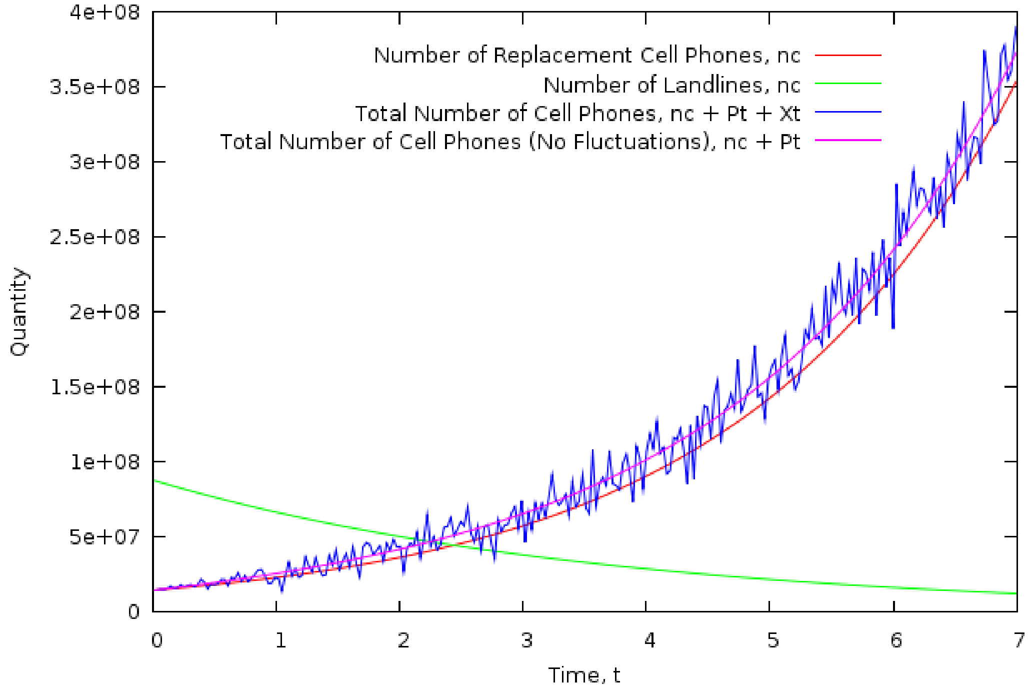


Figure 1. A plot of various quantity functions of our model, specifically the exponential decay of the number of landlines, and the exponential growth of total number of cell phones, without population function, with population function, and with both population function and our modified Weiner Process induced fluctuations

II. Transient Change in Energy Consumption

We are interested in exploring the impact of replacement of landline communication protocols by mobile devices. Our central premise is that the ensued change in energy consumption (assuming other factors impacting energy to be constant) can be expressed as a function of the rate of change of aggregate power consumed by all cell phones in a given geographic entity and that by all landlines in the same region. During the transition, hence,

$$\frac{dE}{dt} = \frac{dE_c}{dt} + \frac{dE_l}{dt} = e_c\left(\frac{dn_c}{dt} + \frac{dP_t}{dt} + X_t\right) + e_l\frac{dn_l}{dt}$$

where E is national energy consumption, E_c is aggregate power consumed by cell phones, E_l is aggregate power used by landline devices, n_c is the total number of cell phones in use in a given geographic locality *contributed by landline replacement*, n_l is the total number of landlines in the same locality, e_c is the power consumed by one cell phone (assumed to be constant for the moment), e_l is the power consumed by a landline phone (assumed to be constant in this work), P_t is the population of the locality at the moment, and X_t is a modified one-dimensional Weiner Process (explained later).

A. *Communication Device Usage Trends*

We now discuss the replacement strategy of landlines by cell phones, and attempt to present a mathematical model of the same. We assume the replacement procedure to involve a deadline policy strategy, with the appropriate governing body imposing a timespan of $\triangle t$ starting from a replacement moment r, after which landline serviced would be removed entirely. Hence, the transition stage would be represented as $r < t < r + \triangle t$, and the steady state after transition is over would be stated as $t > r + \triangle t$. Note that for our purpose of mathematical analysis, $r = t_o = 0$.

A.1 *Landlines*

During the transition, we can model the decline of the total number of landlines in use as an exponential decay function. The choice of exponential decay is viable, since it is best representative of typical ultimatum policy-making (which we assume in this paper for the replacement process), with an initial announcement of transition by the governing body of the transition deadline that results in rapid replacements by most fractions of the population capable enough, following by a steady declining rate of replacement when the lower tier population undertakes replacement (possibly by government aid), and finally, the the tail of the population that is forced to replace or loose communication altogether. Therefore, we can assert that

$$\frac{dn_l}{dt} = -\lambda_l n_l = -\lambda_l A e^{-\lambda t}$$

For $n_l = Ae^{-\lambda_l t}$, A is equal to the number of landlines in use at the moment replacement was initiated, $n_{l,r}$ (estimated by available data, Appendix A), and by assuming that the number of landlines would have quartered midway through the transition period (that is, at $t = r + \frac{\triangle t}{2} = \frac{\triangle t}{2}$), we can solve for λ_l by asserting that

$$n_l\left(\frac{\triangle t}{2}\right) = \frac{n_{l,r}}{4} = n_{l,r}e^{-\lambda_l\left(\frac{\triangle t}{2}\right)}$$

and solving for λ to obtain

$$\lambda_l = \frac{2ln(4)}{\triangle t}$$

Our final model for landline decline is hence re-stated as,

$$\frac{dn_l}{dt} = -\frac{2ln(4)}{\triangle t}n_{l,r}e^{-\frac{2ln(4)}{\triangle t}t}$$

A.2 *Cell Phones Replacement*

Within the same transition period of landline decline, cell phones are assumed to replace landlines by proliferating among m members of a household that previously owned a landline. Hence, in the best possible post-transition scenario,

$$n_c\left(r + \triangle t\right) = n_{l,r}\left(m - m_r\right)$$

where n_c, again, is the number of cell phones contributed to the aggregate count of cell phones in a nation by landline replacement. We can now model the trend towards the above equality in a manner similar to our model of landline decline from the previous section; in the case of cell phones, the model is now that of growth, the absolute value of the rate of which is certainly faster than that of landline decline due to the fact that $m - m_r$ number of cell phones must be provided for each lost landline, where m_r is the number of members that initally possessed a cell phone alongside a common household landline. Our model for exponential growth of cell phone quantity is now,

$$\frac{dn_c}{dt} = \lambda_c n_c = \lambda_c n_{c,r}e^{\lambda_c t}$$

Based on our assumed replacement trend, we can assert that midway through the transition period, again at $t = r + \frac{\triangle t}{2} = \frac{\triangle t}{2}$, the quantity of cell phones accumulated would equal the initial amount added to $\frac{3}{4}n_{l,r}\left(m - m_r\right)$, and hence solve for λ_c by specifying,

$$n_c\left(\frac{\triangle t}{2}\right) = n_{c,r} + \frac{3}{4}n_{l,r}\left(m - m_r\right) = n_{c,r}e^{\lambda_c\left(\frac{\triangle t}{2}\right)}$$

and obtain,

$$\lambda_c = 2\triangle t^{-1}ln\left(\frac{n_{c,r} + \left(m - m_r\right)\left(\frac{3}{4}n_{l,r}\right)}{n_{c,r}}\right)$$

A.3 *Population Growth*

During both the transition and steady state of cell phone proliferation (after replacement has been initiated, $t > r$), the growing population is assumed to contribute new cell phones to the nation, without any addition to the number of landlines. For the United States, or a similar region ("Pseudo US"), we can model the population of growth to be logistic - the exponential stage of growth in this model is visible in current data sets (which we use to fit the logistic function), and the saturation stage is projected for the future with a declining birth rate (increased use of birth control, eduction, and the like), coupled with lower immigration rates (attributed to stricter immigration laws) and steady death rates. As such,

$$\frac{dP_t}{dt} = P_t\left(1 - P_t\right)$$

Since the new population after replacement has been initiated purely contributes cell phones and no landlines, P_t in the above equation can be specified as:

$$P_t = \frac{c}{1 + ae^{-bt}} - P_r$$

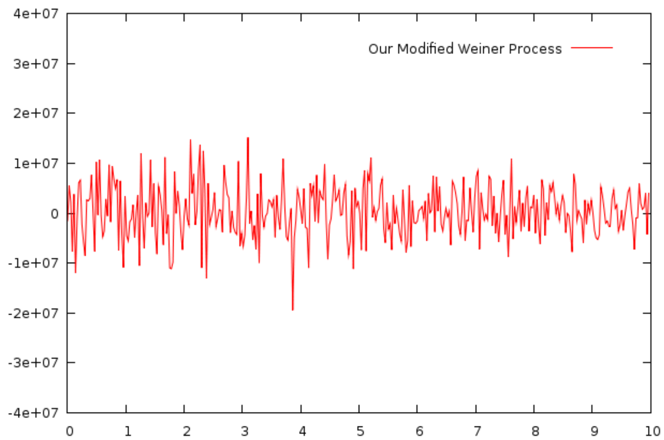
where P_r is the population of the nation the moment replacement was initiated. In the above equation, the parameters a, b, and c are determined using logistic curve fitting (on a Texas Instruments TI-84 calculator) on population data from 1900 (obtained from [21]) to 2050 (projections from [22]) to be

$$\begin{aligned} a &= 1.0646224042617 \\ b &= 0.0179409946970 \\ c &= 636954900.71328 \end{aligned}$$

A.4 *Device Quantity Fluctuations*

Unlike landline phones, which can be conveniently assumed to have an arbitrarily large lifespan and a negligible replacement frequency, mobile devices generally do not demonstrate a robust nature in existence. As such, the number of cell phones in active use in any given time interval is fluctuating rapidly, despite an overal trend of exponential growth in aggregate number. In this work, we attempt to account for these fluctuations through inclusion of brownian motion within our quantity model for cell phones - for this work, we utilize the stochastic process, denoted as the Weiner Process, for modeling fluctuations in rate of change of cell phone quantities entailed by factors such as the unpredictable (and generally brief) mobile device lifespan, attributed to loss from portability, surrounding damage, and general trends in fashion. The Weiner Process can be defined as X_t, specifically,

$$X_t = \mu t + \sigma W_t$$



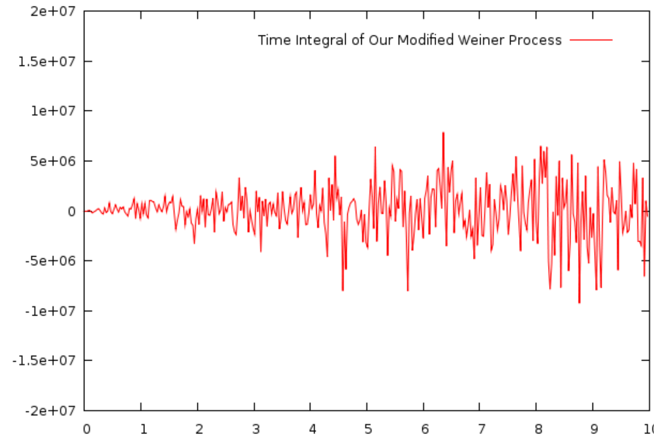


Figure 2. Plots of our modified version of the Weiner Process, $X_t = \sigma W_t$, where $W_t \sim N\left(0, \xi_0 e^{-\frac{ln(2)}{\triangle t}t}\right)$. The left hand side depicts the Weiner Process incorporated into our energy consumption differential equation, while the right hand side plot shows its time integral, approximated using a Riemann Sum.

where μ represents the drift of the process, σ^2 is the variance, and W_t is Standard Brownian Motion that is continuous, is composed of independent increments, and is characterized as $W_t \sim N(0, t)$.

Since we incorporate the Weiner Process as "noise" in our quantity model, we assume $\mu = 0$, to entail only fluctuations about the overall exponential growth trend of mobile devices, developed in Section 2.1.2. Our incorporated fluctuations, as such, are

$$X_t = \sigma W_t$$

where we further modify W_t to be $W_t \sim N(0, \xi_t)$, where $\xi(t)$ is an exponential decay function. We use a decay function for the variance of the rate of change of fluctuations, since in the inital stages of replacement, a significant fraction of the population would be newly exposed to cell phones at a permanent level, and hence more prone to losing or damaging them, whereas, at later stages, especially in the steady state, most of the population will either have a sizable experience to the portable devices, or will be the new population born during or after replacement period, that will have indefinite exposure to cell phones (and hence, less prone to loose or damage them). These latter facts are modeled by our modified Weiner Process in that rate of fluctuations during time periods right after replacement initiation are higher, and steadily relax as time progresses. We specify our exponential decay function as,

$$\xi_t = \xi_0 e^{-\lambda t}$$

and arbitrarily assert that fluctuation intensity would have halved at the end of the replacement timespan, thereby asserting that

$$\xi\left(\triangle t\right) = \frac{\xi_0}{2} = \xi_0 e^{-\lambda(\triangle t)}$$

and realizing $\lambda = \frac{ln(2)}{\triangle t}$, allowing us to specify,

$$\xi_t = \xi_0 e^{-\frac{ln(2)}{\triangle t}t}$$

Our final model for capturing fluctuations is therefore stated as,

$$X_t = \sigma W_t$$

where $W_t \sim N(0, \xi_0 e^{-\frac{ln(2)}{\triangle t}t})$. For our analysis, we arbitrarily assume $\xi_0 = n_{c,r}$, though the parameter certainly has potential for a better specification.

B. Energy Consumption in Communication Devices

For the sake of initial simplification, we momentarily assume the following:

1. Landlines are plugged throughout their lifespan, and minor fluctuations in power consumption resulting from brief power outages, or accidental phone damages, are negligible.
2. Mobile devices need not be plugged in during their entire operating period, but only a fraction of it (during which their batteries are charged) - we assume for the moment that these devices are only charged when battery is completely drained, and for a timespan just enough to fill the battery completely.

Based on the above two assumptions, we can assert that

$$\begin{aligned} e_c &= \tau_c p_c \\ e_l &= \tau_l p_l \end{aligned}$$

where e_c is the total energy consumed by a mobile device in one timestep (assumed to be one year for our analysis), and is equal to the wattage of the device (in kW) multiplied by the fraction of the timestep, τ_c (in hours), that

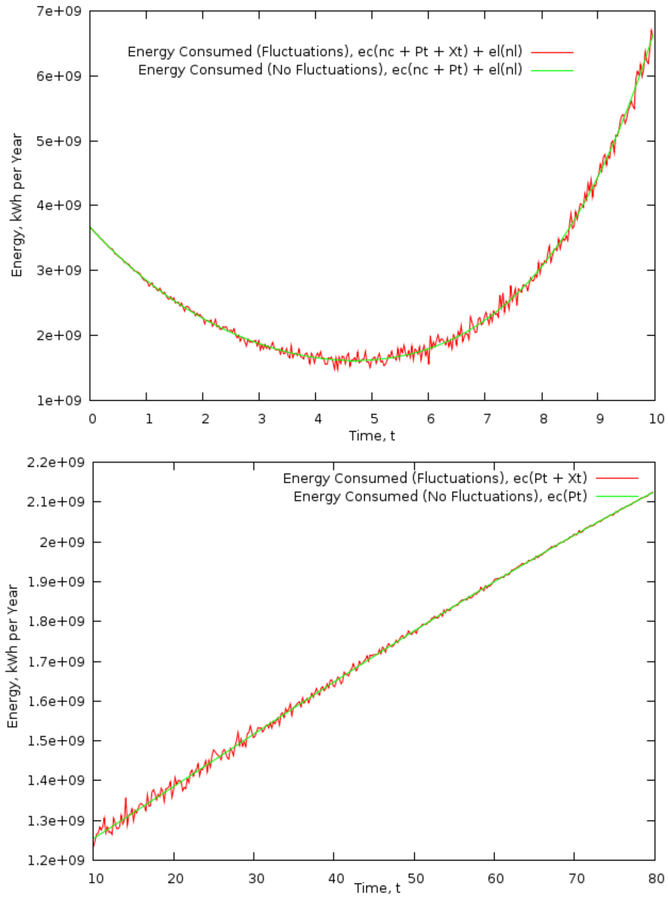


Figure 3. A plot of our energy consumption functions during the transition and the steady state respectively. Note how the function is parabolic in the transition, and has relative minimum at $t = 4.684$, whereas the steady state function is logistic (since the quantity of cell phones during steady state is impacted solely by population growth, which in our model is logistic in trend.

the device is actively consuming electricity from an outlet (or charging device), whereas e_l is assumed to be the total energy consumed by all landline phones in a household in the same timestep, with τ_l and p_l representing energy consumption fraction of timestep and wattage respectively. Note that the unit for e_c and e_l is $\frac{kWh}{year}$. Based on our assumptions, note that τ is equal to the size of our timestep of one year, and hence, equal to 8760 hours per year. τ_c on the other hand, is estimated from available data sources to be 276.776 hours per year, and likewise, p_c and p_l are average wattages of mobile device wall outlet chargers and landline phones respectively, estimated to be 16.67 Watts and 4.67 Watts from available data sources (refer to Appendix B for the estimation process).

C. *Consequences of Replacement on Energy Profile and Optimal Policy*

We can now solve our energy change model during the transition stage to obtain a function for the energy consumed at various time instances during the replacement process. Hence,

$$E_t = e_c \int \left(\frac{dn_c}{dt} + \frac{dP_t}{dt} + X_t \right) + e_l \int \frac{dn_l}{dt}$$

In the above,

$$\int \frac{dn_c}{dt} = n_c = n_{c,r} e^{2\triangle t^{-1} ln\left(n_{c,r} + (m - m_r)\left(\frac{3}{4} n_{l,r}\right)(n_{c,r})^{-1}\right)t}$$

$$\int \frac{dP_t}{dt} = P_t = \frac{c}{1 + ae^{-bt}} - P_r$$

$$\int \frac{dn_l}{dt} = n_l = n_{l,r} e^{-\frac{2ln(4)}{\triangle t} t}$$

and the time integral of our modified Weiner Process is approximated as

$$\int_0^T X_t \approx Y_n = \frac{T}{n} \sum_{k=0}^{n-1} X_{\frac{kT}{n}}$$

This allows us to state that,

$$E_t = e_c \left(n_c + P_t + Y_n \right) + e_l \left(n_l \right)$$

with each of n_c, P_t, Y_n, and n_l specified above. Figure 1 is a graph of the energy consumption function E_t.

Our task now is to find the minimum of this function, which is the time t when $\frac{dE}{dt} = 0$. For simplicity, we omit the Weiner Process and numerically solve for the derivative using a gradient descent program we compose in the Python programming language, and obtain our time, t of lowest energy consumption to be 4.684 years into the transition timespan (based on our policy agenda assumed). What is important to note, however, is not the time periods, or the specific quantities of $n_c + P_t = 136602556.745$ and $n_l = 23987405.945$, but the fact that the *optimal combination of landlines and cell phones in a nation with socioeconomic aspects similar to the United States, based on our model, is one in which the number of cell phones is approximately* 70.125% *higher than the number of landlines.*

III. Energy Consumption in the Steady State

During the steady state, since $\frac{dE_l}{dt} \approx \frac{dn_c}{dt} \approx \frac{dn_l}{dt} \approx 0$, we can assert that the addition of cell phones to the current quantity of the nation is contributed by the growing population, and hence state our energy consumption rate of change with respect to time as,

$$\frac{dE}{dt} = \frac{dE_c}{dt} = e_c \left(\frac{dP_t}{dt} + X_t \right)$$

and likewise, our energy consumption equation as,

$$E_t = e_c \left(P_t + Y_n \right) + e_c \left(n_c \left(\triangle t \right) \right)$$

A plot of the energy consumption during the steady state is shown in Figure 3. It is important to realize that the growth in number of cell phones during the steady state in our model is equivalent to the growth in the population during this time period, since displacement of landlines by new cell phones is now assumed to have halted.

IV. Modeling Mobile Device Energy Consumption

There are two aspects of mobile device energy consumption not previously considered in this work:

1. The fact that cell phones are not as consistently charged as the parameter τ_c in our original cost model depicts. This is observed in typical nightly charging practises, occassional forgetfullness in charging, and overcharging.
2. Various charging protocols, wattages of which are not neccessarily captured by the constant parameter p_c, that only considers wall outlets.

We now refine our cell phone energy consumption model to address these two respective issues, and then utilize these in our energy consumption function to re-predict energy consumption with these factors in consideration, and assess energy loss for the nation as a whole.

A. Charging Frequency and Timespan

We devise a stochastic process for modeling the possibilities of charging practises (in terms of necessity).

$$\hat{\tau}_c(t) = N^{-1}\sum_{k=1}^{N}\sum_{l=1}^{365}\begin{cases}\tau_{c_1} & 0<\varphi_{k,l}<\frac{2}{11}\\ \tau_{c_2} & \frac{2}{11}<\varphi_{k,l}<\frac{5}{11}\\ \tau_{c_3} & \frac{5}{11}<\varphi_{k,l}<\frac{9}{11}\\ \tau_{c_4} & \frac{9}{11}<\varphi_{k,l}<1\end{cases}$$

where N represents the number of repetitions of the stochastic process, $\varphi_{k,l}$ are random numbers from a uniform random distribution, such as $\varphi_{k,l}\sim U(0,1)$, and τ_{c_1}, τ_{c_2}, and τ_{c_3}represent different amounts of charging time undertaken. In order, the charging lengths are 10.5 hours (overcharge), 8 hours (nightly charge), 4 hours (partial charge), and 0 hours (no charge) for a given night (see Appendix B), and their respective probabilities of incidence are $\frac{2}{11}$, $\frac{3}{11}$, $\frac{4}{11}$, and $\frac{2}{11}$.

To compute the timespan yeild of our stochastic process, we implemented it as a Python routine, and computed it for $N = 100,000$ in a CPU time of $6m - 4.906s$. The resultant value for average hours per year that a cell phone gets charged is computed by repitition and averaging of our stochastic process that determines individual behavious on a particular day based on a pre-defined probability distribution - in our simulation, the value is determined to be 2024.149015 hours per year.

B. Charging Protocols

We similarly improve our model for energy consumption by considering the use of various devices for charging a cell phone,

$$\hat{p}_c(t) = \frac{N^{-1}}{365}\sum_{k=1}^{N}\sum_{l=1}^{365}\begin{cases}p_{c_1} & 0<\psi_{k,l}<\frac{1}{4}\\ p_{c_2} & \frac{1}{4}<\psi_{k,l}<\frac{1}{2}\\ p_{c_3} & \frac{1}{2}<\psi_{k,l}<\frac{3}{4}\\ p_{c_4} & \frac{3}{4}<\psi_{k,l}<1\end{cases}$$

where N represents the number of repetitions of the stochastic process, $\psi_{k,l}$ are random numbers from a uniform random distribution, such as $\psi_{k,l}\sim U(0,1)$, and p_{c_1}, p_{c_2}, p_{c_3}, and p_{c_4}represent different means of charging, namely Outlet Charger, USB charger, Car Charger, and Renewable Energy Charger respectively. Using available data, we estimate wattages of each of these charging devices to be 16.67 Watts, 38.33 Watts, 74.33 Watts, and 0 Watts (since no national energy is consumed) in order (see Appendix B). Note that each charging means has an equal chance of getting selected for a particular day of the year for an individual, and the model can certainly be improved to consider differing probabilities for the same.

In a manner similar to that for the stochastic process for charging timespan, we implemented a Python program to compute the stochastic process for charging protocols for $N = 100,000$ in a CPU time of $5m - 23.904s$. Our obtained value for average wattage of cell phone chargers based on various usages probabilities for various devices is 32.3297807 Watts .

C. Energy Wasted by Malpractice

We can now contrast our stochastic models (that account for inefficiencies) for charging timespan and wattages with our previous (efficiency assuming) deterministic models, in order to compute energy wasted in charging cell phones:

Energy loss due to wasteful practices in charging timespans (assuming wall outlet charging only):

$$E_w = \hat{\tau}_c p_c - e_c = \hat{\tau}_c p_c - \tau_c p_c$$

$$\begin{aligned} E_w &= \frac{(2024.149015)(16.67)-(276.776)(16.67)}{365}\\ &= 79.80467989\,Wh\\ &= 0.07980467989kWh\end{aligned}$$

Hence, *0.079 kWh, or 4.6943929347e-5 barrels of oil are wasted per day per person due to wasteful practices in charging timespans.*

Energy loss due to use of various charging protocols (assuming only necessary charging):

$$E_w = \tau_c\hat{p}_c - e_c = \tau_c\hat{p}_c - \tau_c p_c$$

$$E_w = \frac{(276.776)(32.3297807) - (276.776)(16.67)}{365}$$
$$= 11.87466154\, Wh$$
$$= 0.01187466154 kWh$$

Hence, *0.011 kWh, or 6.98509497059e-6 barrels of oil are wasted per day per person due to wasteful practices in charging timespans.*

Energy loss due to both wasteful practices in charging timespans and use of carious charging protocols:

$$E_w = \hat{\tau}_c \hat{p}_c - e_c = \hat{\tau}_c \hat{p}_c - \tau_c p_c$$

$$E_w = \frac{(2024.149015)(32.3297807) - (276.776)(16.67)}{365}$$
$$= 166.6477749\, Wh$$
$$= 0.1666477749 kWh$$

Hence, *0.166 kWh, or 9.80281028824e-5 barrels of oil are wasted per day per person due to wasteful practices in charging timespans.*

V. Model Evaluation

A. Assumptions and Scope

The following are some of the most noteworthy assumptions of our model:

1. We assume a deadline policy strategy for transition from landlines to cell phones; as such, our model fails to fully capture the essence of scenarios where landline decline might naturally occur in favor of cell phones. In the latter case, decay might be more accurately accounted for by a logarithmic decay model, rather than the exponential decay used in our analysis. Similarly, our assertion of quartering of landlines halfway through the transition time period is rather arbitrary, and quite unrepresentative of the real world, where such a trends is heavily dependent on a myriad of other factors.
2. In population based analysis, age groups are not considered, and in the transition stage $m - m_r$ and in the steady state m members are assumed to directly contribute to the total number of cell phones in use, whereas in actuality, a good amount of the $m - m_r$ members in the transition, and m members in the steady state must reach a certain age before which they can maturily handle a mobile communication device.
3. For the steady state, our model predicts that

$$\lim_{t\to\infty} E_t = \lim_{t\to\infty} e_c(P_t + Y_n) + e_c(n_c(\triangle t)) = \infty$$

and so the total number of cell phones after the transition is going to rise *ad infinitum*, whereas in the actual future, new technology may (periodically as usual) displace cell phones as a better medium of communication.
4. Only cell phones and landlines are considered as "communication devices", despite the fact that current technology boasts a myriad of other communication protocols such as VoIP, Instant Messaging, and the like. During displacement, some members of the population may hence choose neither of cell phones or landlines in favor of these other technologies, a fact that is not captured by our model.
5. In cell phone battery life modelling, both stochastic and deterministic, the fact that cell phones are used beyond their normal use of talking (as in text messaging, web browsing, camera, music playback, and the like) is ignored, and could actually lead to our current models underpredicting the energy consumption of cell phones relative to landlines. Landline power consumption modeling is similarly oversimiplified, in that multiple phones might exist per landline, multiple landlines might exist per household, and landlines might not necessarily be plugged in throughout their lifespan.
6. Production and support costs of each of landlines and cell phones are not considered in the energy consumption models - most likely, these costs may very well be higher for cell phones, and may be increasingly so as the devices proliferate among a larger population.

B. Philosophical Aspects

1. Our population models, as well as our energy and device quantity models in general, attempt to extrapolate severely, and predict these variables many decades into the future. These predictions are not necessarily accurate, such as, for instance, a case where a natural catastrophe may devestate population, and severely reduce the number of cell phones in use, something not at all addressed by our model.
2. In our stochastic models for charging habits and charging protocol choice, we attempt to model human behavious and decision making using a random process, which is not necessarily the route of functioning by which the appropriate habits are formed. As such, there is potential for a better model for these cell phone charging habits, such as through a neural network that not only considers probabilities of each type of charging timespan or method, but also other factors such as personal income level, environmental awareness, and the like.

VI. Conclusion

In this work, we considered the scenario of cell phones displacing landlines as means of communication. We began by modeling the transition process and its subsequent steady state, and discussed the impact on the energy profile of the United States, as well as optimal combination of landlines and cell phones to entail the lowest amount of energy usage. Refer to Figure 4 for our final energy consumption models for both the transient and steady state. We then

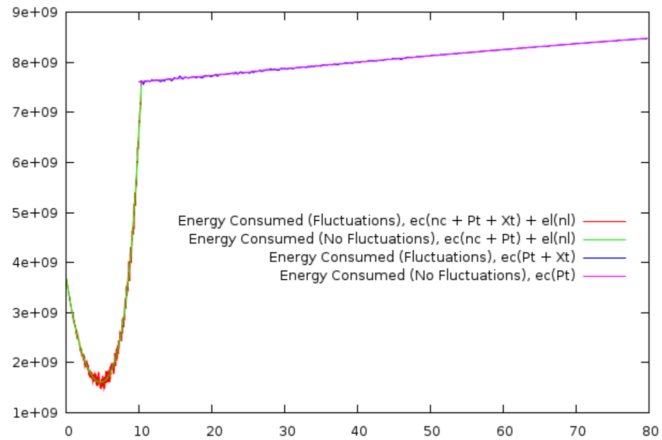


Figure 4. Our final model for energy consumption for both the transition and the steady state

proceeded with improvement of our models of energy consumption for cell phones, taking into account factors such as variance in timespan of nightly charging, and different charge sources. With these refined models, we computed the energy wasted in kWh and Barrels of Oil due to malpractise or wasteful habits in cell phone charging.

References

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Appendix A - *Population and Quantity Estimates*

Average American Household Size = m = 2.59 [25]

Number of OCCUPIED Housing Units = 105,904,641 [25]

Percentage of households with a landline - 59% (Landline + Cell) + 24% (Landline) [24]

Percentage of households with only cell phone - 14% [24]

Assume 25% of household size has cell phone in case of landline and cell phone

Assume deltaT = 10 years

Average Cell Phone Talk time = 0.5 hours per day [23]

In general, it will take 3-4 hours to "rapid charge" a battery to the 80% level, and an additional 8 hours to "slow or trickle" charge a battery to the 100% level [18]

As such,

$$\begin{aligned} n_{l,r} &= (0.59 + 0.24) * 105904641 \\ n_{c,r} &= 0.14 * 105904641 \\ m_r &= 0.25 * 2.59 \end{aligned}$$

Appendix B - *Energy Consumption Estimates*

Estimated from sources [7], [8], [9], [19]:

Cell Phone Model	**Battery Life (standby)**
RIM BlackBerry Pearl 8130	216 h
HTC T-Mobile G1	402 h
LG Glimmer	168 h
Motorola Razr V3	290 h

Cell Phone Model	**Battery Life (talk)**
RIM BlackBerry Pearl 8130	220 min
HTC T-Mobile G1	406 min
LG Glimmer	210 min
Motorola Razr V3	430 min

Average Talk Time = 5.275 h

Average Standby Time = 269 h

Average Talk Time Per Day [23] = 0.5 h

Average Time to Drain:

$$\begin{aligned} 5,275 - 0.5\tau_c &= 0 \\ \tau_c &= 10.55\,days \end{aligned}$$

Estimated from personal owned cell phone chargers:

Charger Model	**Wattage**
LG TA-PO1WR	20W
Motorola FMP52021	15W
Samsung ATADS10JBE	15W

Average Wattage = 16.67 Watts

Estimated from sources [6], [7]:

Landline Phone Model	**Wattage**
Panasonic PQLV30054ZAB	6W
AT&T E5921	5.4W
AT&T EL42108	2.6 W

Average Wattage = 4.67 Watts

Estimated from sources [2], [3], [4]:

Car Charger	**Wattage**
Belkin AC	50 W
Monster Mobile	8 W
Car Adapter	165 W

Average Wattage = 74.33 Watts

Estimated from sources [4], [5]:

USB Charger	**Wattage**
Generic USB	1 W
Generic USB	25 W
Duracell	90 W

Average Wattage = 38.33 Watts

Appendix C - *Timespan.py*

```
import math
import random

tau1 = 10.5
tau2 = 8
tau3 = 4
tau4 = 0

N = 100000

sum = 0

for k in range(0, N):
        for l in range(0, 365):
                r = random.uniform(0,1)
                if r < 2.0/11.00:
                        sum += tau1
                if r > 2.0/11.00 and r <
                    5.0/11.0:
                        sum += tau2
                if r > 5.0/11.00 and r <
                    9.0/11.0:
                        sum += tau3
                if r > 9.0/11.00:
                        sum += tau4

print str(sum / (N ))
```

Appendix D - *Protocol.py*

```
import math
import random

phi1 = 16.67
phi2 = 38.33
phi3 = 74.33
phi4 = 0

N = 100000

sum = 0

for k in range(0, N):
        for l in range(0, 365):
                r = random.uniform(0,1)
                if r < 1.0/4.00:
                        sum += phi1
                if r > 1.0/4.00 and r < 1.0/2.0:
                        sum += phi2
                if r > 1.0/2.00 and r < 3.0/4.0:
                        sum += phi3
                if r > 3.0/4.00:
                        sum += phi4

print str(sum / (N ))
```

Appendix E - *Plots.py*

```
import math
import random

nlr = (.59 + .24) * 105904641
ncr = .14 * 105904641
m = 2.59
mr = .25 * 2.59
```

```
deltaT = 10
sigma0 = 14000000
mu = 0.0
sigma = 0.5
ec = 4.612954188
el = 8.861999997 * ec

def integral(T, n, deltaT, sigma0, mu, sigma):
        sum = 0
        for i in range(0, n-1):
                sum += mu * float(t) + sigma *
                    random.normalvariate(0,
                    sigma0 * math.exp(-t * (math.
                    log(2)/deltaT)))
        return sum * (T/n)

FILE1 = open('nl.dat', 'w')
FILE2 = open('nc.dat', 'w')
FILE3 = open('Pt.dat', 'w')
FILE4 = open('nc_Pt_Xt.dat', 'w')
FILE5 = open('nc_Pt.dat', 'w')
FILE6 = open('nc_nl_Pt_Xt.dat', 'w')
FILE7 = open('nc_nl_Pt.dat', 'w')
FILE8 = open('Xt.dat', 'w')
FILE9 = open('INTXt.dat', 'w')
FILE10 = open('energy_fluctuations.dat', 'w')
FILE11 = open('energy_no_fluctuations.dat', 'w')

for j in range(0, 365):
        t = float(j) * (10.0/365.0)
        nl = nlr * math.exp(-1 * t * ((2 * math.
            log(4))/(deltaT)))
        nc = ncr * math.exp(-1 * t * ((math.log((
            ncr)/(ncr + (m - mr)*(.75 * nlr))))/(
            deltaT)))
        Pt = 636954900.71328 / (1 +
            1.6046224042617 * math.exp
            (-0.0179409946970 4 * t)) +
            636954900.71328 / (1 +
            1.6046224042617 * math.exp
            (-0.01794099469704 * 0))
        dXt = mu * float(t) + sigma * random.
            normalvariate(0, sigma0 * math.exp(-t
             * (math.log(2)/deltaT)))
        Xt = integral(t, 100, deltaT, sigma0,  mu
            , sigma)
        energy_fluctuations = ec * (nc + Pt + Xt)
             + el * (nl)
        energy_no_fluctuations = ec * (nc + Pt) +
             el * (nl)
        FILE1.write(str(t) + ' ' + str(nl))
        FILE1.write('\n')
        FILE2.write(str(t) + ' ' + str(nc))
        FILE2.write('\n')
        FILE3.write(str(t) + ' ' + str(Pt))
        FILE3.write('\n')
        FILE4.write(str(t) + ' ' + str(nc + Pt +
            Xt))
        FILE4.write('\n')
        FILE5.write(str(t) + ' ' + str(nc + Pt))
        FILE5.write('\n')
        FILE6.write(str(t) + ' ' + str(nc + nl +
            Pt + Xt))
        FILE6.write('\n')
        FILE7.write(str(t) + ' ' + str(nc + nl +
            Pt))
        FILE7.write('\n')
        FILE8.write(str(t) + ' ' + str(dXt))
        FILE8.write('\n')
        FILE9.write(str(t) + ' ' + str(Xt))
        FILE9.write('\n')
        FILE10.write(str(t) + ' ' + str(
            energy_fluctuations))
        FILE10.write('\n')
        FILE11.write(str(t) + ' ' + str(
            energy_no_fluctuations))
        FILE11.write('\n')

FILE1.close()
FILE2.close()
FILE3.close()
FILE4.close()
FILE5.close()
FILE6.close()
FILE7.close()
FILE8.close()
FILE9.close()
FILE10.close()
FILE11.close()
```

Appendix F - *GNUPlot Plot Commands*

```
plot 'nc.dat' t 'Number of Replacement Cell
    Phones, nc' with lines, 'nl.dat' t 'Number of
     Landlines, nc' with lines, 'nc_Pt_Xt.dat' t
    'Total Number of Cell Phones, nc + Pt + Xt'
    with lines, 'nc_Pt.dat' t 'Total Number of
    Cell Phones (No Fluctuations), nc + Pt' with
    lines

plot 'energy_fluctuations.dat' t 'Energy Consumed
     (Fluctuations), ec(nc + Pt + Xt) + el(nl)'
    with lines, 'energy_no_fluctuations.dat' t '
    Energy Consumed (No Fluctuations), ec(nc + Pt
    ) + el(nl)' with lines,  'nl.dat' t 'Number
    of Landlines, nc' with lines, 'nc_Pt_Xt.dat'
    t 'Total Number of Cell Phones, nc + Pt + Xt'
     with lines, 'nc_Pt.dat' t 'Total Number of
    Cell Phones (No Fluctuations), nc + Pt' with
    lines

plot 'nc.dat' t 'Number of Replacement Cell
    Phones, nc' with lines, 'nl.dat' t 'Number of
     Landlines, nc' with lines, 'nc_Pt_Xt.dat' t
    'Total Number of Cell Phones, nc + Pt + Xt'
    with lines, 'nc_Pt.dat' t 'Total Number of
    Cell Phones (No Fluctuations), nc + Pt' with
    lines

plot 'energy_fluctuations_steady.dat' t 'Energy
    Consumed (Fluctuations), ec(Pt + Xt)' with
    lines, 'energy_no_fluctuations_steady.dat' t
    'Energy Consumed (No Fluctuations), ec(Pt)'
    with lines

plot 'Pt_Xt_steady.dat' t 'Total Number of Cell
    Phones (Fluctuations), Pt + Xt' with lines, '
    Pt_steady.dat' t 'Total Number of Cell Phones
    , Pt' with lines
```