Global Warming

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Abstract

The greenhouse effect caused by gradual increase of the carbon dioxide content of the Earth's atmosphere is an important topic in modelling due to its general impact on the public. We consider a given set of data for the Earth's temperature difference from 1980 for particular years. We then derive an empirical model from this data set, and use it to predict the approximate year for the rise to approach seven degrees above the 1980 value. We culminate with a discussion of the importance of this value, and the effectiveness of our model in predicting it.

1 Introduction

The greenhouse effect is caused by the an accumulation of greenhouse gases such as carbon dioxide in the Earth's atmosphere. In the recent decades, scientists have observed a trend quite alarming to the climate-focused population - the amount of carbon dioxide in the atmosphere is steadily increasing at a growing rate, and if a certain level is reached, the accompanying increase in temperature could reach a level that could lead to a meltdown of the polar caps, causing drastic consequences for humans [1]. We derive a model for a set of data for temperature differences since 1860 in the Earth's atmosphere, in order to have the capability to present an approximate predict for the year in which a seven degress Celcius difference from 1860 values should arise, assuming all other factors stay constant. This value is important, since it is conjectured that this is an approximately critical value to initiate a breakdown of the polar ice sheets.

2 Empirical Model

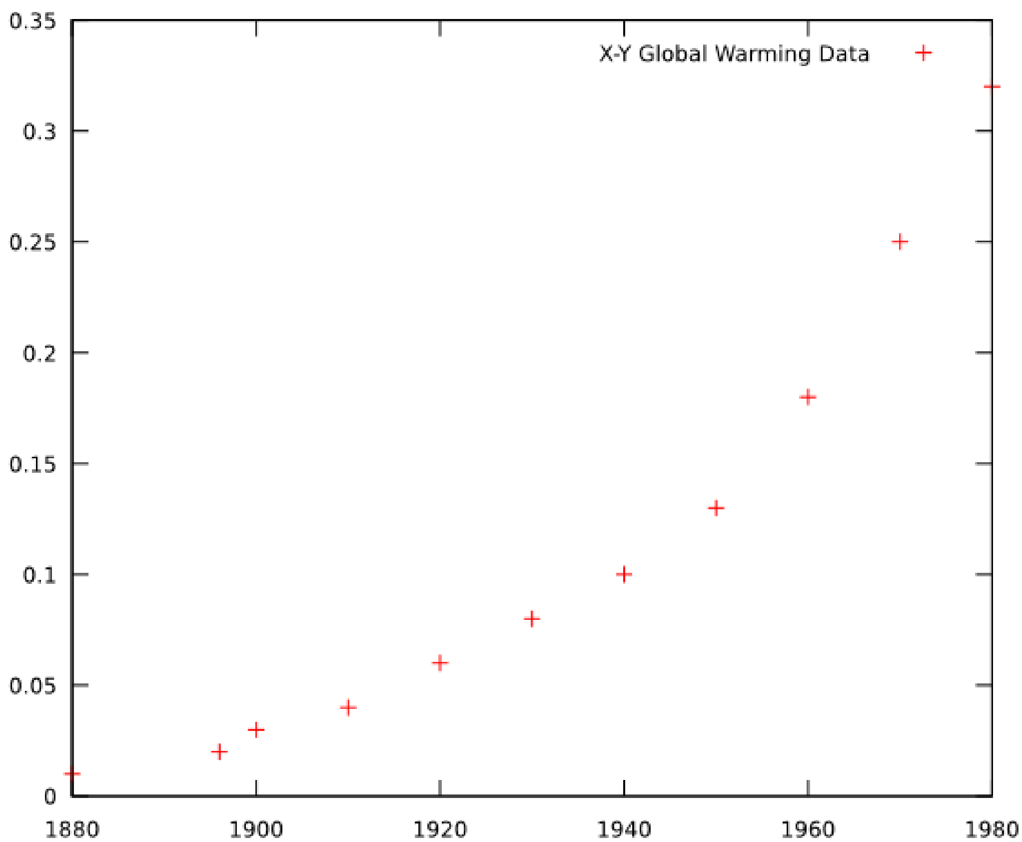
We begin by tabulating and plotting the given data for the temperature difference values for various years compared to 1980. The following shows this tabulation of the data:

Year	Temperature Difference from 1860
1880	0.01
1896	0.02
1900	0.03
1910	0.04
1920	0.06
1930	0.08
1940	0.1
1950	0.13
1960	0.18
1970	0.25
1980	0.32

Issuing the following plot command to GNUPlot® ,

```
set term svg
set output "xygraph.svg"
plot "global_warming_raw_data.dat" title 'X-Y Global Warming Data'
   with points
```

we get the following as the graphical plot for the data

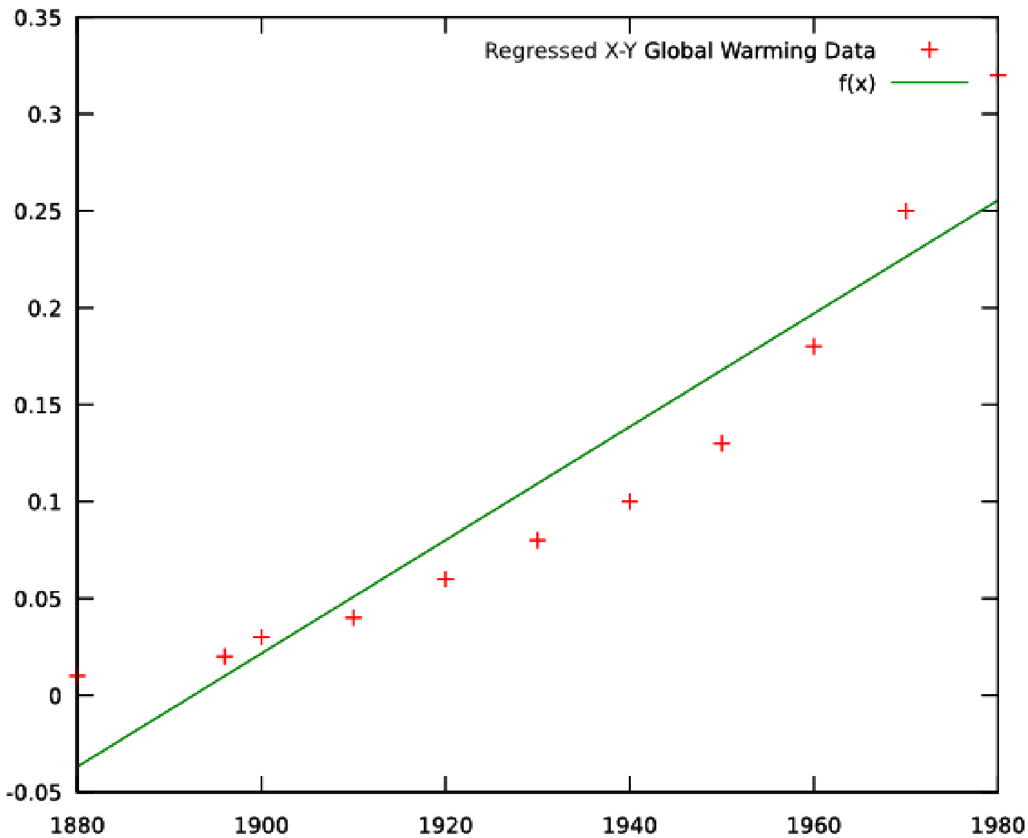


Intuitively, we can conjecture by observation that this data is not going to encouter a good linear regression fit directly. To confirm, we instruct GNUPlot® to fit a linear line of the form $y = ax + b$

into the above data, and present statistics about the regression. The commands issued for defintion, fitting, and plotting of the line amidst the original data scatter plot are as follows

```
set term svg
set output "/home/mukarram/fitted_xygraph.svg"
f(x) = a*x + b
fit f(x) "global_warming_raw_data.dat" using 1:2 via a,b
plot "global_warming_raw_data.dat" title 'Regressed X-Y Global
   Warming Data' with points,f(x)
```

to obtain the graph



as well as the following values

Parameter Variables	Final Values after Fitting	Asymptotic Standard Error
a	0.00292385	11.99% ± 0.0003506
b	-5.53371	12.23% ± 0.677

and statistics for the fit (refer to Appendix A for the detailed fit log)

Degrees of Freedom	9
Root-Mean Squared Error of Residuals	0.03602
Variance of Residuals	0.00129744
R-Squared	0.885397220

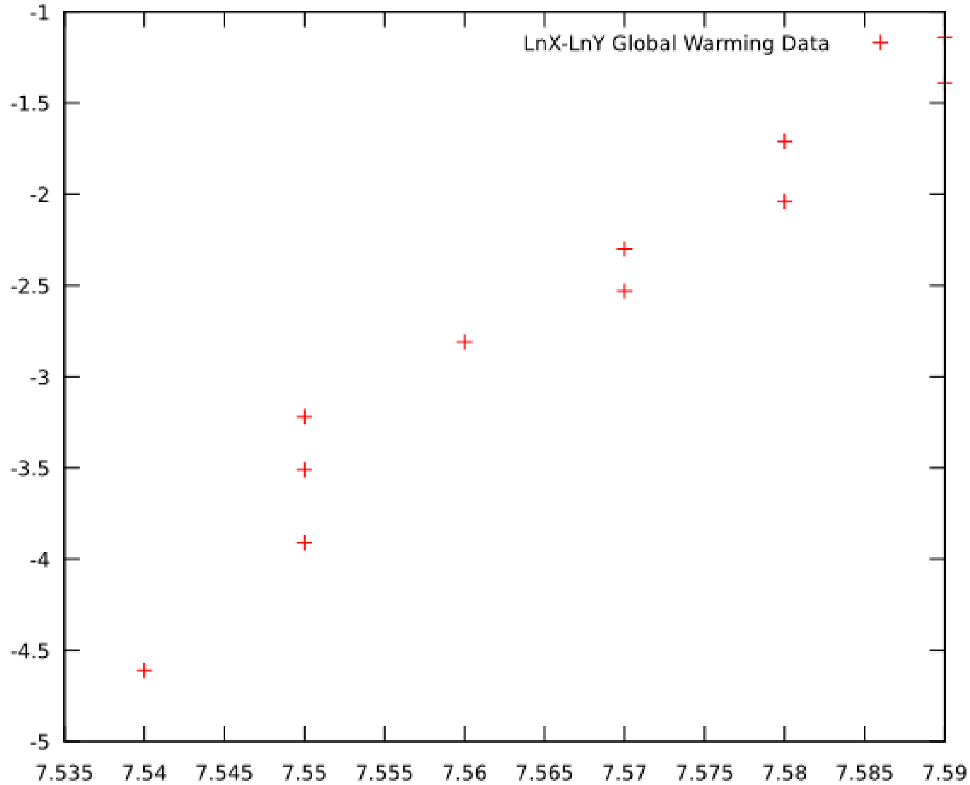
The rather low value for R^2 and the observably non-linear appearance of the trend of the data prompted us to perform manipulations in order to seek a better linear fit for the data. We hence attempted the same procedure for the untreated data on a data set that was derived by calculating the natural logarithm of both the time period and temperature difference values (it is important to realize that we do attempt a non-linear fit of the original data in order to confirm the type of mathematical trend it actually displays - this is done in the next section). The following is our 'log-log' data in a tabular form

Year (Natural Logarithmic)	Temperature Difference from 1860 (Natural Logarithmic)
7.54	-4.61
7.55	-3.91
7.55	-3.51
7.55	-3.22
7.56	-2.81
7.57	-2.53
7.57	-2.3
7.58	-2.04
7.58	-1.71
7.59	-1.39
7.59	-1.14

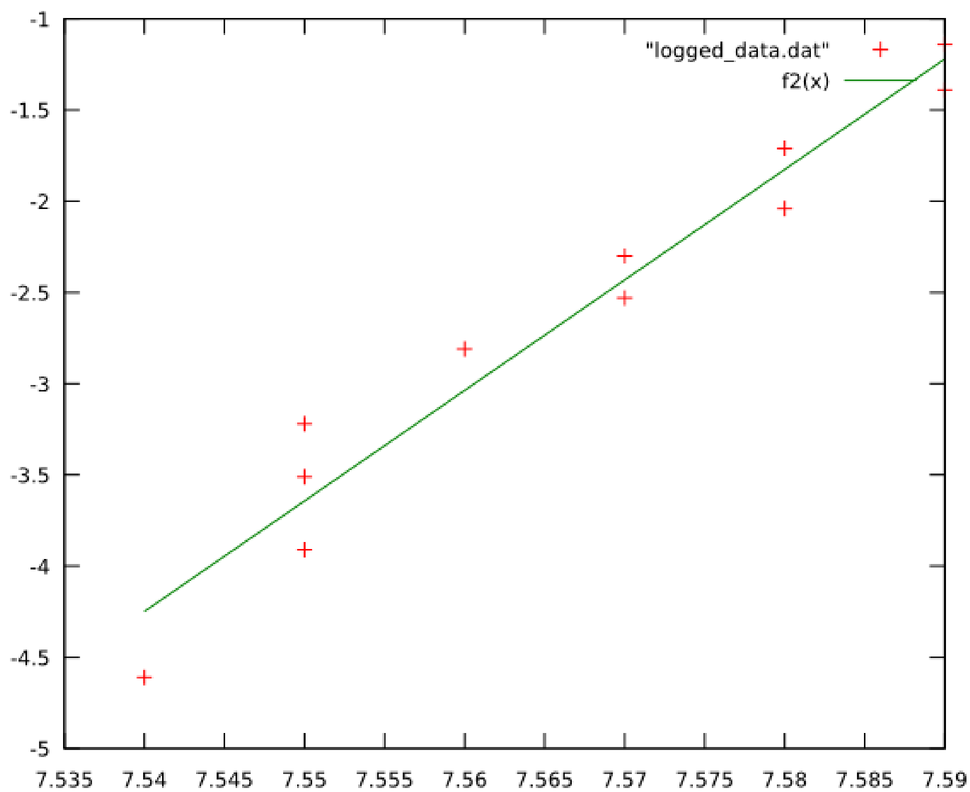
We can issue similar commands as we did for the previous plot and fit process

```
set term svg
set output "logxlogygraph.svg"
plot "logged_data.dat" title 'LnX-LnY Global Warming Data' with
   points
set output "/home/mukarram/fitted_xygraph.svg"
f2(x) = a*x + b
fit f2(x) "logged_data.dat" using 1:2 via a,b
plot "logged_data.dat" title 'Regressed LogX-LogY Global Warming
   Data' with points,f2(x)
```

in order to obtain the following plot,



the following fitted best-fit line,



and the following fit values

Parameter Variables	Final Values after Fitting	Asymptotic Standard Error
a	60.5744	7.528% ± 4.56
b	-460.98	7.485% ± 34.5

and statistics

Degrees of Freedom	9
Root-Mean Squared Error of Residuals	0.252024
Variance of Residuals	0.0635163
R-Squared	0.983957373

It is quite certain that the value for R^2 is significantly higher than for the linear regression of the data not treated with the 'log-log' procedure. Hence, the linear regression of the 'log-log' data is the best model in this case for the given data.

3 Utilization for Prediction

Our first step before we start using the model for direct prediction of temperature differences is to derive a final expression for the relationship of time period and temperature difference in context of our fitting procedure in the previous section. We begin by assuming a power law relationship in the given global warming data set, and so use the values for a and b obtained in the previous section to derive the final power law expression for our model, in order to be able to predict the year in which the temperature difference will be seven degrees higher than 1860. The power law relationship is

$$y = ax^b$$

If we take the natural logarithm of both sides, as we did for the given data set in the previous section, we obtain

$$lny = ln(ax^b)$$

Using the property of logarithms that evaluation of the logarithm of two terms being multiplied is equivalent to addition of the logarithms of the individual terms, we can write the above equation as

$$lny = ln(a) + bln(x)$$

and for clarity, we can assert that $X = lnx$ and $Y = lny$, changing the previous equation by substitution into

$$Y = lna + bX$$

In this relationship, b is the slope, which, from the previous section, should equal a (when we defined a as the slope during the regression), and vice versa for lna in the above equation. Hence, the above equation's variables are defined as:

$$lna = -460.98$$

$$b = 60.5744$$

Noting that $a = e^{-460.98}$, we can now plug a and b into the original power law relationship $y = ax^b$ to obtain our final model for the global warming data set

$$y = e^{-460.98}x^{60.5744}$$

Setting y equal to 7, since we want temperature difference to be seven degrees from 1860,

$$7 = e^{-460.98}x^{60.5744}$$

and solving for x, we obtain the year in which global temperature would be different from 1860 temperatures by seven degrees, based on our model, to be **2084**.

4 Limitations

Our empirical model for the global warming data set certainly has several shortcomings. One of the most important ideas to consider is that by utilizing our model for predicting the year for a seven degree difference, we are extrapolating quite a significant distance from the whereabouts of the original data, the range of which is 0.01 to 0.32. This extrapolation is dangerous because the linear regression model will generally only give the right prediction for values arbitrarily close to the range of the original values, and at values larger by several order of the original ones, we just cannot avoid the arising of inaccuracies for the prediction of the empirical model. Also, the model is only dependant upon the validity of the data set - if the data points are erroneous, our model follows the same route of error. Similarly, we have no explanation whatsoever for the trend; all that was done in the above procedures was mathematical regression of a given data set, out of which no theoritical explanation can be derived for observed trends.

5 Conclusion

We began by considering a data set for temperature differences of the Earth's atmosphere from 1860 global temperature. We then proceeded with attempting a linear regression of the data, and concluded that a 'log-log' technique allows the best fit for a linear curve. We then derived a power law relationship, $y = e^{-460.98}x^{60.5744}$ from the fitting process, and solved it to come to a realization that a seven degree difference from 1860 temperatures would occur in 1984. We culminated by a discussion of the limitations of our models, including our attempted extrapolation, dependancy on data reliability, and lack of trend explanation.

References

[1] Climate Change. http://www.epa.gov/climatechange/

Appendix A: Fit Log for Raw Global Warming Data

```
*******************************************************************************
Tue Oct 14 23:52:18 2008


FIT:    data read from "global_warming_raw_data.dat" using 1:2
        #datapoints = 11
        residuals are weighted equally (unit weight)

function used for fitting: f(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 4.10453e+07       delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda   : 1365.28

initial set of free parameter values

a               = 1
b               = 1

After 8 iterations the fit converged.
final sum of squares of residuals : 0.011677
rel. change during last iteration : -1.28503e-08

degrees of freedom    (FIT_NDF)                        : 9
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.03602
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.00129744

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 0.00292385       +/- 0.0003506    (11.99%)
b               = -5.53371         +/- 0.677        (12.23%)


correlation matrix of the fit parameters:

               a      b
a              1.000
b             -1.000  1.000
```

```
*******************************************************************************
Tue Oct 14 23:56:09 2008


FIT:    data read from "logged_data.dat" using 1:2
        #datapoints = 11
        residuals are weighted equally (unit weight)

function used for fitting: f2(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 101.739           delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda   : 5.39676

initial set of free parameter values

a               = 0.00292385
b               = -5.53371

After 7 iterations the fit converged.
final sum of squares of residuals : 0.571647
rel. change during last iteration : -1.9305e-13

degrees of freedom    (FIT_NDF)                        : 9
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.252024
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.0635163

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 60.5744          +/- 4.56         (7.528%)
b               = -460.98          +/- 34.5         (7.485%)


correlation matrix of the fit parameters:

               a      b
a              1.000
b             -1.000  1.000
```

```
*******************************************************************************
Wed Oct 15 02:34:36 2008


FIT:    data read from "global_warming_raw_data.dat" using 1:2
        #datapoints = 11
        residuals are weighted equally (unit weight)

function used for fitting: f(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 4.10453e+07       delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda   : 1365.28

initial set of free parameter values

a               = 1
b               = 1

After 8 iterations the fit converged.
final sum of squares of residuals : 0.011677
rel. change during last iteration : -1.28503e-08

degrees of freedom    (FIT_NDF)                        : 9
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.03602
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.00129744

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 0.00292385       +/- 0.0003506    (11.99%)
b               = -5.53371         +/- 0.677        (12.23%)


correlation matrix of the fit parameters:

               a      b
a              1.000
b             -1.000  1.000
```

Appendix B: Fit Log for Log-Log Global Warming Data

```
*******************************************************************************
Tue Oct 14 23:52:18 2008


FIT:    data read from "global_warming_raw_data.dat" using 1:2
        #datapoints = 11
        residuals are weighted equally (unit weight)

function used for fitting: f(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 4.10453e+07       delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda   : 1365.28

initial set of free parameter values

a               = 1
b               = 1

After 8 iterations the fit converged.
final sum of squares of residuals : 0.011677
rel. change during last iteration : -1.28503e-08

degrees of freedom    (FIT_NDF)                        : 9
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.03602
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.00129744

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 0.00292385       +/- 0.0003506    (11.99%)
b               = -5.53371         +/- 0.677        (12.23%)


correlation matrix of the fit parameters:

               a      b
a               1.000
```

```
b                -1.000  1.000


*******************************************************************************
Tue Oct 14 23:56:09 2008


FIT:    data read from "logged_data.dat" using 1:2
        #datapoints = 11
        residuals are weighted equally (unit weight)

function used for fitting: f2(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 101.739           delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda   : 5.39676

initial set of free parameter values

a               = 0.00292385
b               = -5.53371

After 7 iterations the fit converged.
final sum of squares of residuals : 0.571647
rel. change during last iteration : -1.9305e-13

degrees of freedom    (FIT_NDF)                        : 9
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.252024
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.0635163

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 60.5744          +/- 4.56         (7.528%)
b               = -460.98          +/- 34.5         (7.485%)


correlation matrix of the fit parameters:

               a      b
a               1.000
b              -1.000  1.000
```

```
***********************************************
Wed Oct 15 02:34:36 2008


FIT:    data read from "global_warming_raw_data.dat" using 1:2
        #datapoints = 11
        residuals are weighted equally (unit weight)

function used for fitting: f(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 4.10453e+07       delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda   : 1365.28

initial set of free parameter values

a               = 1
b               = 1

After 8 iterations the fit converged.
final sum of squares of residuals : 0.011677
rel. change during last iteration : -1.28503e-08

degrees of freedom    (FIT_NDF)                        : 9
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.03602
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.00129744

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 0.00292385       +/- 0.0003506    (11.99%)
b               = -5.53371         +/- 0.677        (12.23%)


correlation matrix of the fit parameters:

               a      b
a               1.000
b              -1.000  1.000
```

```
**********************************************
Wed Oct 15 03:03:03 2008


FIT:    data read from "global_warming_raw_data.dat" using 1:2
        #datapoints = 11
        residuals are weighted equally (unit weight)

function used for fitting: f2(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 0.011677          delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda   : 1365.28

initial set of free parameter values

a               = 0.00292385
b               = -5.53371

After 1 iterations the fit converged.
final sum of squares of residuals : 0.011677
rel. change during last iteration : 0

degrees of freedom    (FIT_NDF)                        : 9
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.03602
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.00129744

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 0.00292385       +/- 0.0003506    (11.99%)
b               = -5.53371         +/- 0.677        (12.23%)


correlation matrix of the fit parameters:

               a      b
a              1.000
b             -1.000  1.000
```

```
*******************************************************************************
Wed Oct 15 03:04:17 2008


FIT:    data read from "logged_data.dat" using 1:2
        #datapoints = 11
        residuals are weighted equally (unit weight)

function used for fitting: f2(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 101.739           delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda   : 5.39676

initial set of free parameter values

a               = 0.00292385
b               = -5.53371

After 7 iterations the fit converged.
final sum of squares of residuals : 0.571647
rel. change during last iteration : -1.9305e-13

degrees of freedom    (FIT_NDF)                        : 9
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.252024
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.0635163

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 60.5744          +/- 4.56         (7.528%)
b               = -460.98          +/- 34.5         (7.485%)


correlation matrix of the fit parameters:

               a      b      
a               1.000 
b              -1.000  1.000 


*******************************************************************************
```

```
Wed Oct 15 03:06:14 2008


FIT:    data read from "logged_data.dat" using 1:2
        #datapoints = 11
        residuals are weighted equally (unit weight)

function used for fitting: f2(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 1395.74            delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                  limit for stopping : 1e-05
 lambda   : 5.39676

initial set of free parameter values

a               = 1
b               = 1

After 7 iterations the fit converged.
final sum of squares of residuals : 0.571647
rel. change during last iteration : -2.41992e-13

degrees of freedom    (FIT_NDF)                        : 9
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.252024
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.0635163

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 60.5744          +/- 4.56         (7.528%)
b               = -460.98          +/- 34.5         (7.485%)


correlation matrix of the fit parameters:

               a      b
a               1.000
b              -1.000  1.000
```